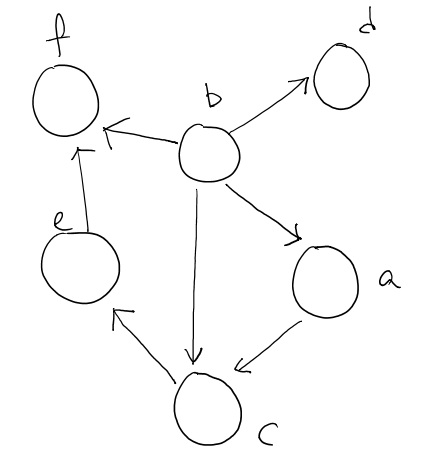
**ECE521 Assignment 4**

Written by

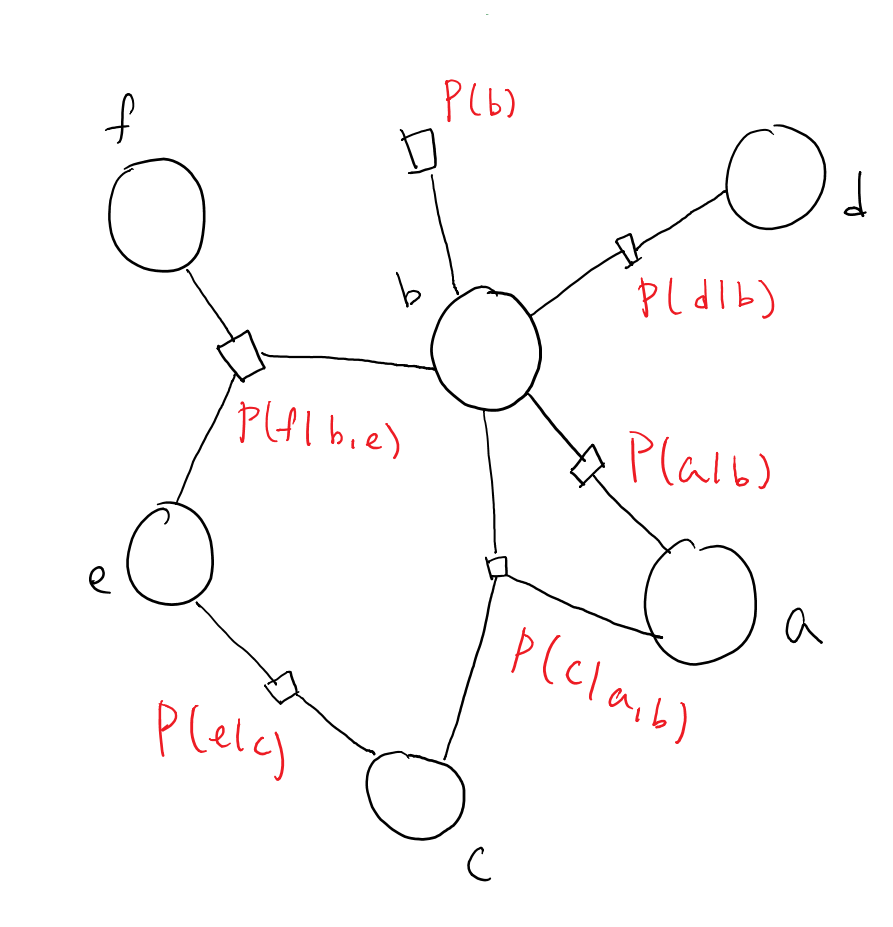
Winston Wong (1001614853)

Date: 2017-04-08

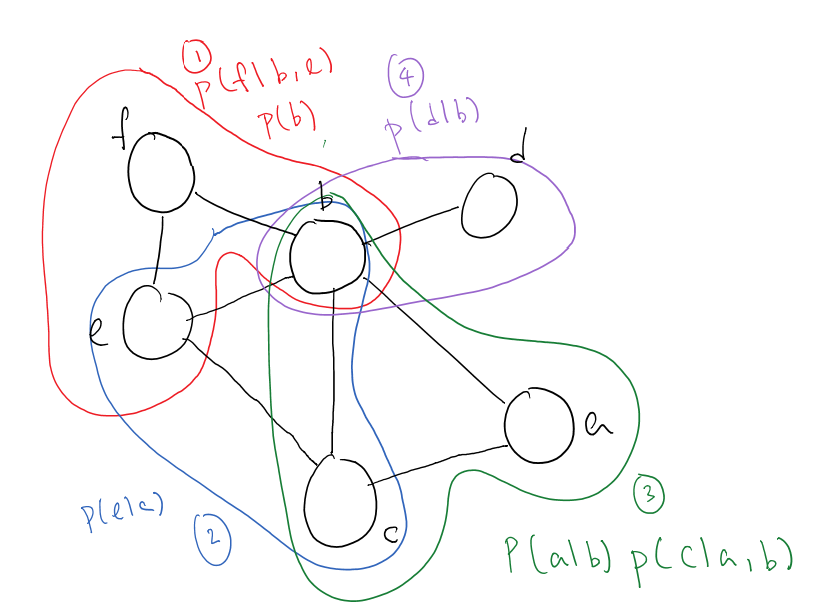
**1.1.1**



**1.1.2**



**1.1.3**



**1.2.1.1a)**

|  |  |  |
| --- | --- | --- |
|  | Factor Graph  (given in question) | Bayesian Network |
| Graph |  |  |
|  | False | False |
|  | False | False |
|  | False | False |
|  | False | False |
|  | False | False |
|  | False | False |

Conditional Probabilities in terms of factors using sum-product algorithm

**1.2.1.1b)**

**NOTE**: A Bayesian Network representation of the factor graph below is impossible if

This is because the factors and indicates that b and c have no parent but indicates there’s conditional dependence between the two. Thus either b is the parent or c is the parent, which is a contradiction.

However, if we relax our assumption, that is, assume , Then the simplified factor graph can be converted to the BN as follows.

|  |  |  |
| --- | --- | --- |
|  | Factor Graph  (given in question) | Bayesian Network |
| Graph |  |  |
|  | False | False |
|  | False | False |
|  | False | False |
|  | False | False |
|  | False | False |
|  | False | False |

The conditional Probabilities in terms of factors are computed as follows.

**1.2.1.2a)**

|  |  |  |
| --- | --- | --- |
|  | Factor Graph  (given in question) | Markov Random field |
| Graph |  |  |
|  | False | False |
|  | False | False |
|  | False | False |
|  | False | False |
|  | False | False |
|  | False | False |

**1.2.1.2b)**

|  |  |  |
| --- | --- | --- |
|  | Factor Graph  (given in question) | Markov Random field |
| Graph |  |  |
|  | False | False |
|  | False | False |
|  | False | False |
|  | False | False |
|  | False | False |
|  | False | False |

**1.2.2.1**

|  |  |  |
| --- | --- | --- |
|  | Markov Random field  (given in question) | Factor graph |
| Graph |  |  |
|  | False | False |
|  | False | False |
|  | False | False |
|  | False | False |
|  | False | False |
|  | False | False |
|  | False | False |
|  | False | False |
|  | True | True |
|  | False | False |
|  | False | False |
|  | False | False |
|  | False | False |
|  | False | False |
|  | True | True |
|  | False | False |
|  | False | False |
|  | False | False |

**1.2.2.2**

No. First given that and are true for the given MRF, there cannot be any connections between the diagonal vertices in BN. Thus we look at the BNs without diagonal connection, with only 4 edges connecting the adjacent vertices Moreover, the combinations made by the 4 edges in the graph can be boiled down to 3 by Symmetry, as follows. It is obvious that adjacent nodes are always conditionally dependent. We investigate conditional independence between diagonal vertices.

|  |  |  |
| --- | --- | --- |
| 2 “head to tail” arrows | 3 “head to tail arrows” | 4 head to tail arrows |
|  |  |  |
| FALSE | FALSE | NOT POSSIBLE SINCE BN CONTAINS CYCLE |
| FALSE | FALSE |
| FALSE | FALSE |
| TRUE | TRUE |

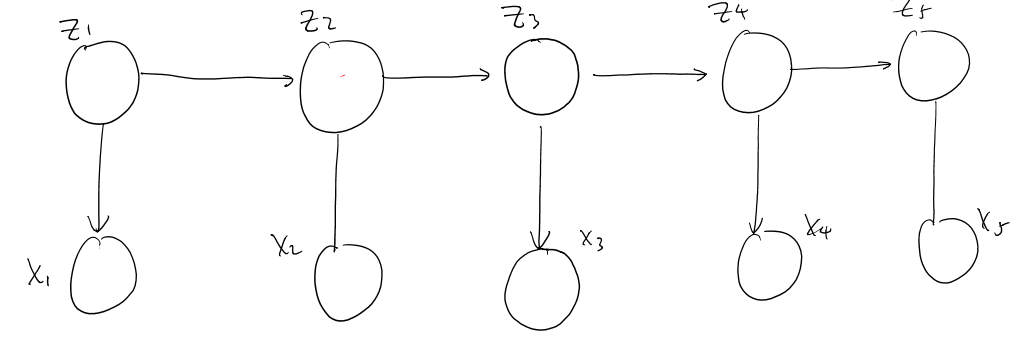
In both cases, there exists one conditional independence between the diagonal vertices.

**1.3.1**

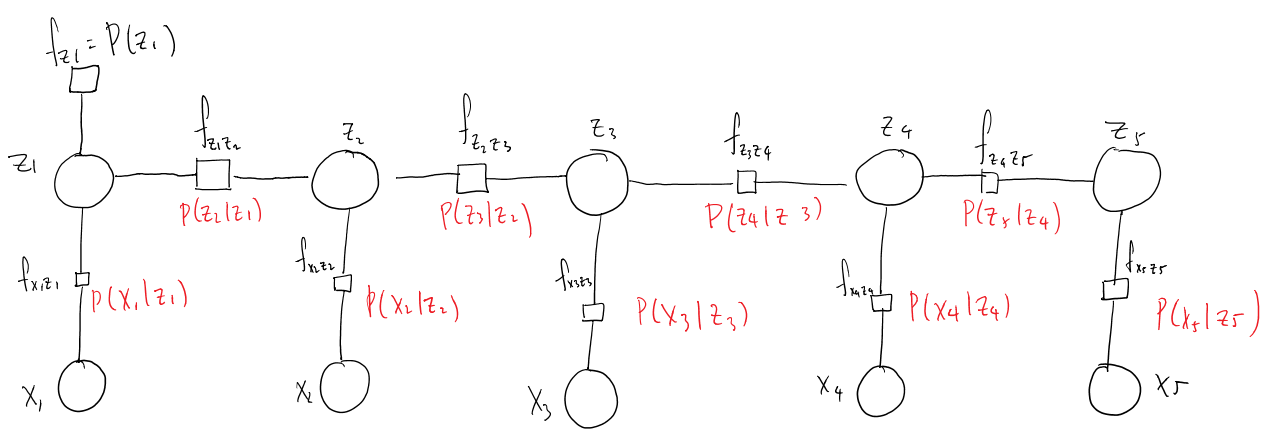
**1.3.2**

|  |  |
| --- | --- |
|  | FALSE |
|  | TRUE |
|  | TRUE |
|  | FALSE |
|  | TRUE |
|  | FALSE |

**3.1.1**



**3.1.2**



**3.2.1**

**3.3.1 NOTE: denotes element-wise product**

Where

Moreover

Thus

**3.3.2 NOTE: denotes element-wise product**

Notice, here we are summing over (NOT over as in the previous question). This is equivalent to multiplying not